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EXPERIMENTAL RESEARCH ON SCATTERING

University of Maryland
Department of Physics and Astronomy
College Park, Maryland 20742

21 June 1979

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Some aspects of the theory of coherent scattering are discussed, theoretically, together with results of experiments to verify the theory.

The cross sections are calculated for scattering by an ensemble, for a number of cases. These include having several kinds of particles in the final state, and inelastic scattering in which external fields provide the quantum states required for conservation of energy and momentum.

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INTRODUCTION

The scattering of light has been studied for several centuries. The discovery of the electron, and the phenomena of radioactivity made available a number of particles for new types of scattering experiments.

The experiments gave considerable information concerning the structure of nucleii, atoms, and compounds.

Scattering experiments have continued to be a major source of information about interactions and particles. The classical and quantum theory of scattering have been very well developed, for both single scatterers and ensembles of scatterers.

For an ensemble of scatterers there are processes in which the total scattering cross section is proportional to the number of scatterers, and processes in which the total cross section is proportional to the square of the number of scatterers. In the latter case the process is referred to as a coherent process. Under certain conditions large effects may occur because of constructive interference.

Existing theory and reports of experiments in the open literature provide no examples of large total scattering cross sections for the scattering of neutrinos. Indeed, there are no published data on coherent effects in neutrino scattering.

It is the purpose of this investigation to explore, theoretically and experimentally, a number of neutrino scattering processes, in order to discover those which give very large coherent total scattering cross sections.

THEORY OF SOME COHERENT SCATTERING PROCESSES

In the scattering of light the wave amplitude is given by an object

$$R = \sum_a \sum_t \left(\frac{V_{it}V_{tf}}{F_1(E)} + \frac{V_{it}V_{tf}}{F_2(E)} \right)_a \tag{1}$$

The letter a refers to the a^{th} scatterer, the letter i refers to the initial state, the letter f refers to the final state, the letter t refers to some transient (intermediate) quantum state of the scatterer. V_{it} is a matrix element of the scattering potential given by

$$V_{it} = \int \psi_i^* V \psi_t d^3x \tag{2}$$

In (2) ψ_i and ψ_t are wavefunctions of the scatterer, in (1) $F_1(E)$ and $F_2(E)$ are functions of the energies of the light quanta and scatterer. The sum over a will involve a random phase for each scatterer a if the initial state ψ_i is different from the final state ψ_f. This happens in Raman scattering. However, if $\psi_f = \psi_i$ then the phase factor associated with each scatterer will cancel out for the product $V_{it}V_{tf}$ since it is then

$$\int \psi_i^* V \psi_t d^3x \int \psi_t^* V \psi_i d^3x \tag{3}$$

The important issue is that if the final state is identical with or in fact very close to the initial state, the random phase associated with a particular scatterer will not affect the amplitude. The resulting amplitude may approach the product of the number of scatterers and the amplitude contributed by one scatterer. The total scattering cross section may then be proportional to the square of the number of scatterers.

A NEW PRINCIPLE OF COHERENT SCATTERING

For an incident particle interacting with many scatterers, the cross section is usually computed in the following way. For each scatterer the amplitude is calculated, assuming that the scatterer is acting independently. And for each scatterer treated in this way the energy and momentum are conserved. For the entire ensemble of scatterers the resulting amplitude is a sum over all scatterers taking the phase shifts and retardation effects into account. The requirements of coherence and energy momentum conservation are very strict and as a result the coherent scattering cross sections calculated in this way are often very small. There are situations in physics where a large number of particles may interact, as a single entity. For example an elementary particle may excite the normal modes of a solid.

Here it is proposed that an entire ensemble of scatterers can collectively and coherently scatter a single incident particle such as a neutrino. The energy momentum conservation relations are assumed to apply to the entire ensemble, acting as one entity. The conservation laws apply to individual particle interactions only if <u>all</u> interactions of a given scatterer with <u>all</u> other particles are included. Each scatterer final state is nearly the same as the original state. It is essential to arrange experiments so that appropriate initial and final states are available to satisfy these requirements. Later, some experimental data will be presented, indicating that this new principle of coherent scattering may be valid for some experiments.

It should be noted that all of the calculations must be done with appropriate collective particle scatterer quantum states.

We propose to apply these ideas to the scattering of neutrinos in a number of ways. First, consider the Feynman graphs, Figures 1 and 2. In Figure 1 the solid line represents an electron scattering a neutrino at A, which changes its energy momentum. At B the electron interacts with the electromagnetic field. Figure 2 represents the same process as Figure 1 except that the interactions with electromagnetic and neutrino fields occur in the opposite order from Figure 1. Thus an incoming neutrino produces an outgoing photon by interaction with electrons.

For such processes the S matrix is given by

$$S=\frac{eG}{\sqrt{2}}\int[\bar{\psi}_{ef}(x)\not{A}(x)S_F(x-y)\gamma^{\alpha}(1+\gamma_5)\psi_{\nu i}(y)\bar{\psi}_{\nu f}(y)\gamma_{\alpha}(1+\gamma_5)\psi_{ei}(y)$$

$$+\bar{\psi}_{ef}(x)\gamma^{\alpha}(1+\gamma_5)\psi_{\nu i}(x)\bar{\psi}_{\nu f}(x)\gamma_{\alpha}(1+\gamma_5)S_F(x-y)\not{A}(y)\psi_{ei}(y)]d^4x\,d^4y \tag{4}$$

$\bar{\psi}_{ef}$ is the final state electron creation operator, ψ_{ei} is the initial state electron annihilation operator, A is the Maxwell 4 potential, the γ's are the gamma matrices appropriate for the Dirac equation, $\not{A} = \gamma_{\mu}A^{\mu}$, S_F is the Feynman propagator, ψ_{ν} are the neutrino field operators.

Let all particles for the moment be described by free particle wavefunctions. S_F is written as a Fourier integral over 4 momentum space. The integration (4) is then carried out to give for N scatterers with final state identical with initial state

$$S = \sum_{j=1}^{j=N}(2\pi)^4\mu_j\delta_4(P_{ph} - \Delta P_{\nu}) \tag{5}$$

μ_j is the matrix element for the j^{th} scatterer, P_{ph} is the 4 momentum of the photon, ΔP_{ν} is the difference of the neutrino initial and final state 4 momenta. (5) is then employed to compute the differential cross section, assuming identical initial and final scatterer states

$$d\sigma = (2\pi)^4\,(N\mu)^2\,\delta_4(P_{ph} - \Delta P_{\nu})\,\pi_k\,\frac{d^3P_k}{(2\pi)^3}. \tag{6}$$

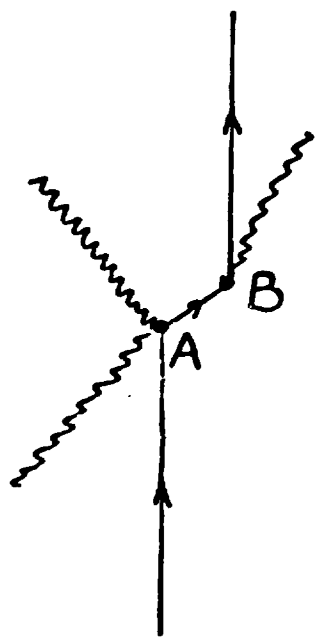

FIGURE 1

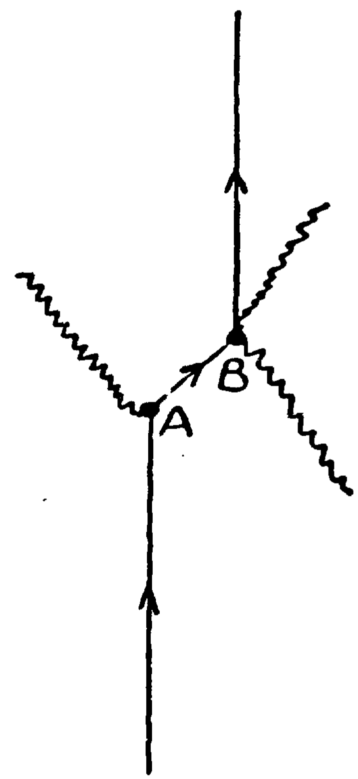

FIGURE 2

In (6) the continued product is over all final state particles. One of the quantities appearing in the integration of (6) is the solid angle into which the neutrino is scattered.

Since each scatterer is assumed to return exactly to its initial state, all of the change in energy momentum of the neutrino is taken by the photon. Under these conditions, the cross section for this process will be shown to be zero.

Let the incoming neutrino be moving in the z direction with momentum $P_{\nu zi}$. Let the outgoing neutrino have momenta $P_{\nu zo}$, $P_{\nu xo}$, and let the photon have momenta P_{phxo}, P_{phzo} then

$$P_{\nu xo} + P_{phxo} = 0 \tag{7}$$

$$P_{\nu zi} = P_{\nu zo} + P_{phzo} \tag{8}$$

(7) and (8) are statements of momentum conservation. Energy conservation requires

$$P_{\nu zi} = P_{\nu zo} + P_{phzo} = \sqrt{P_{\nu zo}^2 + P_{\nu xo}^2} + \sqrt{P_{phzo}^2 + P_{phxo}^2} \tag{9}$$

(9) can be satisfied only if $P_{\nu xo} = P_{phxo} = 0$ and the scattering is strictly in the forward direction. Therefore, the solid angle into which the neutrino is scattered is zero. The cross section is zero.

The statement is often made in textbooks that in Rayleigh scattering of light the initial and final states of the scatterer are identical. Equations (7),(8), and (9) are also applicable to Rayleigh scattering and suggest that the initial and final states cannot in fact be identical. Clearly, they must be sufficiently near each other that the random phase shifts discussed earlier do not destroy coherent effects. In the case of atmospheric Rayleigh scattering from gas molecules, the situation is believed to be the following.

The scattering is by electrons bound in molecules. The wavefunction consists of a part associated with the internal electronic state and a part associated with the translation and or rotation. In the scattering of light the electron returns to its initial electronic state, but transfers some momentum to the nucleus by the coulomb interaction. Cooperative coherent scattering appears possible in which a number of molecules absorb the momentum and scatter one photon. A very small amount of energy is associated with the momentum transfer. As a result there is only a very small phase shift, the entire final quantum state is nearly but not identical with the initial quantum state, and a range of scattering angle is possible for the photon. Many atoms can cooperatively scatter one photon.

For our further discussions, we will consider cases where the initial and final wavefunction are very nearly but not exactly the same.

EVALUATION OF MATRIX ELEMENTS FOR NEUTRINO ELECTRON PHOTON SCATTERING

The matrix element μ appearing in (5) may be written in terms of the electron and neutrino spinors U_e , U_ν and the Fourier transform of the free electron propagator S_F, as

$$\mu = \frac{eG}{\sqrt{2}} \Big[\bar{U}_{ef} \not{A} S_F \gamma^\alpha (1+\gamma_5) U_{\nu i} \bar{U}_{\nu f} \gamma_\alpha (1+\gamma_5) U_{ei} + U_{ef} \gamma^\alpha (1+\gamma_5) U_{\nu i} \bar{U}_{\nu f} \gamma_\alpha (1+\gamma_5) S_F \not{A} U_{ei} \Big] \tag{10}$$

In many scattering calculations, it is customary to square (10) and then sum over all spin states obtaining expressions which are the trace of a long product of matrices. In this calculation, it is essential to sum over all scatterers first. It is helpful also to separate (10) into terms which contain spin operators and terms which do not contain spin operators. For evaluation of (10), we will assume that U_{ef} and U_{ei} represent electrons at rest and $U_{\nu i}$

represents a neutrino propagating in the z direction. For γ^α and $1+\gamma^5$ we choose

$$\gamma^\alpha = \begin{vmatrix} 0 & a_\alpha \\ b_\alpha & 0 \end{vmatrix} \tag{11}$$

$$1+\gamma^5 = 2 \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \tag{12}$$

Momentum conservation requires in the first term of (10)

$$\bar{U}_{ef} \not{A}\, S_F = \bar{U}_{ef} \not{A} \frac{1}{\not{p}_e + \not{p}_{ph} - m} = \frac{\bar{U}_{ef} \not{A} \not{p}_{ph}}{2 p_e \cdot p_{ph}} \tag{13}$$

and for the second term of (10)

$$S_F \not{A}\, U_{ei} = \frac{1}{\not{p}_e - \not{p}_{ph} - m} = \frac{\not{p}_{ph} \not{A}\, U_{ei}}{2\, p_e \cdot p_{ph}} \tag{14}$$

Let $\hat{n}_{pol}$ be a unit vector in the photon polarization direction. Then $\not{A} = \bar{\gamma}\cdot\hat{n}_{pol} A$. Let $\hat{n}_{ph}$ be a unit vector in the photon propagation direction and let $\hat{n}_{\Delta\nu}$ be a unit vector in the direction of the difference of the neutrino initial and final state momenta. Let the 4 component spinors U be expressed in terms of 2 components spinors χ, η by

$$U = \begin{vmatrix} \chi \\ \eta \end{vmatrix} \tag{15}$$

then

$$\mu = \frac{eG\sqrt{2}}{m} \begin{vmatrix} 0 & \chi^\dagger_{ef} \end{vmatrix} \Bigg[(\bar{\gamma}\cdot\hat{n}_{pol} A)(\gamma_0 + \hat{n}_{\Delta\nu}\cdot\bar{\gamma}) \begin{vmatrix} 0 & 0 \\ b^\alpha & 0 \end{vmatrix} \begin{vmatrix} \chi_{\nu i} \\ \eta_{\nu i} \end{vmatrix} \begin{vmatrix} \chi^\dagger_{\nu f} & \eta^\dagger_{\nu f} \end{vmatrix} \begin{vmatrix} b_\alpha & 0 \\ 0 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} 0 & 0 \\ b^\alpha & 0 \end{vmatrix} \begin{vmatrix} \chi_{\nu i} \\ \eta_{\nu i} \end{vmatrix} \begin{vmatrix} \chi^\dagger_{\nu f} & \eta^\dagger_{\nu f} \end{vmatrix} \begin{vmatrix} b^\alpha & 0 \\ 0 & 0 \end{vmatrix} (\gamma_0 + \hat{n}_{ph}\cdot\bar{\gamma})(\bar{\gamma}\cdot\hat{n}_{pol} A) \Bigg] \begin{vmatrix} \chi_{ei} \\ 0 \end{vmatrix} \tag{16}$$

Consider the object

$$\begin{vmatrix} o & o \\ b_\alpha & o \end{vmatrix} \begin{vmatrix} \chi_{\nu i} \\ \eta_{\nu i} \end{vmatrix} \begin{vmatrix} \chi_{\nu f}^\dagger & \eta_{\nu f}^\dagger \end{vmatrix} \begin{vmatrix} b^\alpha & o \\ o & o \end{vmatrix} = \begin{vmatrix} o & o \\ b_\alpha \chi_{\nu i} \chi_{\nu f}^\dagger b^\alpha & o \end{vmatrix} \tag{17}$$

$$\text{Let } B = b_\alpha \chi_{\nu i} \chi_{\nu f}^\dagger b^\alpha \tag{18}$$

Employing (17) and (18) in (16) gives

$$\mu = \frac{ieG/2}{m} \begin{vmatrix} o & \chi_{ef}^\dagger \end{vmatrix} \left[(\bar{\gamma}\cdot\hat{n}_{pol}A)(\gamma_o + \hat{n}_{\Delta\nu}\cdot\bar{\gamma}) \begin{vmatrix} o & o \\ B & o \end{vmatrix} + \begin{vmatrix} o & o \\ B & o \end{vmatrix} (\gamma_o + \hat{n}_{ph}\cdot\bar{\gamma})(\bar{\gamma}\cdot\hat{n}_{pol}A) \right] \begin{vmatrix} \chi_{ei} \\ o \end{vmatrix} \tag{19}$$

(19) contains the terms

$$\bar{\gamma}\cdot\hat{n}_{pol}A(\gamma_o + \hat{n}_{\Delta\nu}\cdot\bar{\gamma}) = \hat{n}_{pol}\cdot\hat{n}_{\Delta\nu}\, A + \bar{\gamma}\cdot\hat{n}_{pol}A\gamma_o + \sum_{ij} \gamma_i\gamma_j n_{pol\,i} n_{\Delta\nu j}\, A (\text{with } i \neq j) \tag{20}$$

$$(\gamma_o + \hat{n}_{ph}\cdot\bar{\gamma})(\bar{\gamma}\cdot\hat{n}_{pol}A) = \hat{n}_{ph}\cdot\hat{n}_{pol}A + \gamma_o A\bar{\gamma}\cdot\hat{n}_{pol} + \sum_{ij} \gamma_i\gamma_j\, n_{phi} n_{pol\,j} A (\text{with } i \neq j) \tag{21}$$

If unpolarized scatterers are employed, only the spin independent terms will contribute significantly to the sum, because terms containing spin operators are spin averages. It is also important to note that the spins of the scatterers may or may not change in consequence of the scattering.

We assume

$$\chi_{\nu i} = \begin{vmatrix} 1 \\ o \end{vmatrix} \text{ for neutrinos and } \begin{vmatrix} o \\ 1 \end{vmatrix} \text{ for antineutrinos}$$

$$\chi_{\nu f} = \begin{vmatrix} \alpha_f \\ \beta_f \end{vmatrix} \quad \chi_{ei} = \chi_{ef} = \begin{vmatrix} \alpha_e \\ \beta_e \end{vmatrix} \tag{22}$$

In (22) we have assumed $\chi_{ei} \approx \chi_{ef}$, the lack of exact equality will be important in the phase space integral.

Then

$$\sum_{electrons} \chi^{\dagger}_{ef} B\, \chi_{ei} = -2N\alpha^*_{\nu f} \quad \text{for neutrinos}$$

$$\sum_{electrons} \chi^{\dagger}_{ef} B\, \chi_{ei} = -2N\beta^*_{\nu f} \quad \text{for antineutrinos}$$

For summation of (10) over all scatterers we have for the terms not containing the spin operators

$$\Sigma\mu = \frac{2\sqrt{2}\ ieG}{m}\ N \begin{bmatrix} \beta_{\nu f}\text{(antineutrinos)} \\ \text{or} \\ \alpha_{\nu f}\text{(neutrinos)} \end{bmatrix} \left[\hat{n}_{\Delta\nu}\cdot\hat{n}_{pol} + \hat{n}_{ph}\cdot\hat{n}_{pol} \right] \frac{f(\theta,\phi)}{\sqrt{E_{ph}}} \left[1+\frac{A_e^2}{\langle A_{vac}^2 \rangle} \right]^{1/2} \tag{23}$$

$$\text{Let}\quad K_{pH} = i2\sqrt{2} \begin{bmatrix} \beta_{\nu f}\text{(antineutrinos)} \\ \text{or} \\ \alpha_{\nu f}\text{(Neutrinos)} \end{bmatrix} \left[\hat{n}_{\Delta\nu}\cdot\hat{n}_{pol} + \hat{n}_{ph}\cdot\hat{n}_{pol} \right] \frac{f(\theta,\phi)}{\sqrt{E_{ph}}} \left[1+\frac{A_e^2}{\langle A_{vac}^2 \rangle} \right]^{1/2} \tag{24}$$

and

$$\Sigma\mu = \frac{eGNK_{ph}}{m\sqrt{E_{ph}}} \tag{25}$$

N is the number of scatterers, $f(\theta,\phi)$ is a directivity factor including retardation effects, m is the electron mass, E_{ph} is the photon energy.

In(23) and (24) the factor $\left[1+\frac{A_e^2}{\langle A_{vac}^2 \rangle} \right]$ takes into account the effects of spontaneous emission and other fields described by A_e. $\langle A_{vac}^2 \rangle$ is the vacuum expectation value of A^2 per mode. Thus if A_e is due to black body radiation, we have

$$\frac{A_e^2}{\langle A_{vac}^2 \rangle} = \frac{2}{\left[e^{\frac{\hbar\omega}{kT}} - 1 \right]} \tag{26}$$

We note then that the coherently emitted photons may be the result of neutrino induced spontaneous emission or stimulated emission as a result of the neutrinos, and electromagnetic field A_e.

It is also important to note that (25) will result if we consider the entire ensemble of scatterers as one particle since $e \rightarrow Ne$; $G \rightarrow NG$ $m \rightarrow Nm$. The entire ensemble scatters <u>one</u> neutrino.

NEUTRINO ELECTRON PHONON SCATTERING

A neutrino may scatter as a result of interactions with electrons and nucleons in a solid. Consider first electron neutrino phonon scattering.

A phonon displaces nucleii relative to the electrons and produces a polarization. The energy of interaction between electrons and the displaced nucleii is

$$H_{INT} = \int \frac{\rho_e(r)\rho_{nu}(r')d^3x d^3x'}{|\bar{r} - \bar{r}'|} \tag{27}$$

ρ_e is the electron charge density given by

$$\rho_e = -e\psi^+\psi \tag{28}$$

ρ_{nu} is the charge density of the displaced nucleii. Let $\bar{q}$ be the particle displacement associated with phonons. Then the polarization $\bar{P}$ may approach the value

$$\bar{P} = \frac{ZeN}{V}\bar{q} \tag{29}$$

Here Z is the nuclear charge number and $\frac{N}{V}$ is the number of nucleii per unit volume. From electrostatics, it is known that

$$\rho_{nu} = \nabla \cdot \bar{P} \tag{30}$$

The relations (27) - (30) then lead to

$$H_{INT} = \frac{Ze^2N}{V} \int \frac{\psi^\dagger\psi(r)\nabla\cdot q(r')d^3x d^3x'}{|\bar{r} - \bar{r}'|} \tag{31}$$

because of screening the potential is not of infinite range. In a certain level of approximation, we may make the replacement

$$\frac{1}{|\bar{r} - \bar{r}'|} \simeq a^2\delta_3 (\bar{r} - \bar{r}') \tag{32}$$

In (32) a is the linear dimension of a unit cell. (31) and (32) then lead to

$$H_{INT} = \frac{Ze^2 Na^2}{V} \int \psi^\dagger\psi \nabla\cdot\bar{q}\, d^3x \tag{33}$$

The phonon displacement $\bar{q}$ is written as the Fourier series

$$\bar{q} = \Sigma \frac{\bar{k}}{|k|} \sqrt{\frac{1}{2\rho_m\omega(k)V}} \left[b_k e^{i(\bar{k}\cdot\bar{r}-\omega t)} + b_k^+ e^{-i(\bar{k}\cdot\bar{r}-\omega t)}\right] \tag{34}$$

Here ρ_m is the mass density, V is again the volume, k is the wavenumber. The divergence of q is required and is evaluated as

$$\nabla\cdot\bar{q} = \sum_k \frac{1}{v_s} \sqrt{\frac{\omega}{2V\rho_m}} \left[b_k e^{i[\bar{k}\cdot\bar{r}-\omega t]} + b_k^+ e^{-i[\bar{k}\cdot\bar{r}-\omega t]}\right] \tag{35}$$

the phonon field operator ϕ is defined as

$$\phi = \sum_k \sqrt{\frac{\omega}{2V}} \left[b_k e^{i[\bar{k}\cdot\bar{r} - \omega t]} + b_k^+ e^{-i[\bar{k}\cdot\bar{r} - \omega t]}\right| \tag{36}$$

(33), (35) and (36) then give

$$H_{INT} = g \int \psi^\dagger\phi\psi d^3x \tag{37}$$

with

$$g = \frac{Ze^2Na^2}{Vv_s\sqrt{\rho_m}} \tag{38}$$

Corresponding to the neutrino electron photon process of equation (4) we have the neutrino electron phonon process with S matrix given by

$$S = \frac{gG}{\sqrt{2}} \iint \Big[\bar{\psi}_{ef}(x)\gamma_o \phi(x) S_F(x-y)\gamma^\alpha (1+\gamma_5)\psi_{\nu i}(y)\, \bar{\psi}_{\nu f}(y)\, \gamma_\alpha (1+\gamma_5)\psi_{ei}(y) \tag{39}$$

$$+ \bar{\psi}_{ef}(x)\gamma^\alpha (1+\gamma_5)\psi_{\nu i}(x)\bar{\psi}_{\nu f}(x)\gamma_\alpha (1+\gamma_5) S_F(x-y)\gamma_o \phi(y)\psi_{ei}(y)\Big]\, d^4x\, d^4y$$

The matrix elements are then

$$\mu = \frac{gG}{\sqrt{2}} \Big[U^{\dagger}_{ef}\phi_p \quad S_F\gamma^\alpha (1+\gamma_5)\, U_{\nu i}\bar{U}_{\nu f}\, \gamma_\alpha (1+\gamma_5) U_{ei}$$

$$+ U_{ef}\, \gamma^\alpha (1+\gamma_5) U_{\nu i}\bar{U}_{\nu f}\, \gamma_\alpha (1+\gamma_5) S_F \gamma_o \phi_p \quad U_{ei} \Big] \tag{40}$$

In (40) ϕ_p is the part of the phonon field operator which creates a phonon with 4 momentum p_{pho}. For standing waves the space part of p_{pho} is zero (40) contains in the first term

$$S_F = \frac{1}{\not{p}_{ei} + \Delta\not{p}_\nu - m} \tag{41}$$

p_{ei} is the initial state electron momentum, Δp_ν is the difference of the initial and final state neutrino momenta. (41) may be written as

$$\frac{\not{p}_{ei} + \Delta\not{p}_\nu + m}{2p_{ei}\cdot\Delta p_\nu} \tag{42}$$

the second term of (40) contains the factor

$$S_F = \frac{1}{\not{p}_{ei} - \not{p}_{pho} - m} \tag{43}$$

(43) is written as

$$\frac{\not{p}_{ei} - \not{p}_{pho} + m}{-2p_{ei}\cdot p_{pho}} \tag{44}$$

These relations enable evaluation of (40) as

$$\mu = \frac{gG}{\sqrt{2}} \left| o \quad \psi^{\dagger}_{ef} \right| \left[\left| \begin{matrix} o & o \\ \beta & o \end{matrix} \right| \left(\frac{2+\frac{\Delta p_{\nu}}{m}}{2\,\Delta p_{\nu}} \right) - \left(\frac{v_{e}[2+\frac{\Delta E_{phon}}{m}]}{\Delta E_{phon}} \right) \left| \begin{matrix} o & o \\ \beta & o \end{matrix} \right| \right] \left| \begin{matrix} \psi_{ei} \\ o \end{matrix} \right| \tag{45}$$

For a process in which the initial and final electron states are identical the change in neutrino energy momentum is equal to that carried away by the phonon, and (45) will sum to zero. Such a process would have an incredibly small cross section even if (45) had the value given only by the second term.

On the other hand it appears possible for the entire ensemble of scatterers to absorb the neutrino energy-momentum, in which case $c\Delta p_{\nu} >> E_{phon}$ and the second term of (45) is the primary contribution. Each scatterer has a final state very close but not identical to the initial state. In this case we may expect for the summation of (45) over N scatterers

$$\mu = NgG \left| \begin{matrix} \beta_{uf}\text{(antineutrinos)} \\ \text{or} \\ \alpha_{\nu f}\text{(neutrinos)} \end{matrix} \right| \sqrt{E_{phon}} \; f(\theta,\phi) \left[1 + \frac{\langle\phi_e^2\rangle}{\langle\phi^2\rangle_o} \right]^{1/2} \tag{46}$$

the factor $1 + \frac{\langle\phi_e^2\rangle}{\langle\phi^2\rangle_o}$ takes into account effects of a phonon field ϕ_e which may be present and the zero point fluctuations $\langle\phi^2\rangle_o$.

PHASE SPACE INTEGRALS

Calculation of the scattering cross section requires the evaluation of an integral over the phase space of all outgoing particles. The invariant integral for n outgoing particles is

$$N_n = \int \frac{d^3p_1 d^3p_2 \ldots . d^3p_n \delta(p_f - p_i)\delta(E_f - E_i)}{E_1 E_2 \ldots . E_n} \tag{47}$$

In (47) p_f and p_i are the total final state and initial state three momenta, respectively. E_f and E_i are the total energies. For some purposes and in a certain level of approximation , the ensemble of scatterers may be treated as a single particle. For the evaluation of (47) with 3 outgoing particles, the integral over p_3 is carried out and this eliminates the three space momentum delta function. d^3p_1 is replaced by $d\Omega_1 p_1^2 dp_1$ with a similar replacement for d^3p_2 obtaining

$$N_3 = \int \frac{d\Omega_1 d\Omega_2 p_1^2 dp_1 p_2^2 dp_2 \delta(E_f - E_i)}{E_1 E_2 E_3} \tag{48}$$

the energy relation

$$E^2 = p^2 + m^2 \tag{49}$$

gives $EdE = pdp$ and (48) becomes

$$N_3 = \int \frac{d\Omega_1 d\Omega_2 p_1 p_2 dE_1 dE_2 \delta(E_f - E_i)}{E_3} \tag{50}$$

Let θ_{12} be the angle between the final state momenta of particles 1 and 2. $d\Omega_1 = 2\pi d(\cos\theta_{12})$ and this leads to

$$N_3 = 2\pi \int \frac{d\Omega_1 dE_1 dE_2 p_1 p_2 d(\cos\theta_{12})\delta(E_f - E_i)}{E_3} \tag{51}$$

The calculation is performed in the center of mass coordinate system, with

$$p_3^2 = p_1^2 + 2\bar{p}_1 \cdot \bar{p}_2 + p_2^2 \tag{52}$$

partial differentiation of (52) leads to

$$p_3 dp_3 = p_1 p_2 d(\cos\theta_{12}) \tag{53}$$

Up to this point the evaluation of (47) has followed the standard procedures. Now let 1 refer to the neutrino, let 2 refer to the photon and let 3 refer to the energy and momentum of the ensemble of scatterers.

Suppose the ensemble of scatterers consists of particles weakly coupled to each other with individual exchanges of energy and momenta uncorrelated. Let the average kinetic energy of each scatterer be $<E^2>_{kei}$. Then

$$p_3^2 = \sum_{i=1}^{N} p_i^2 = N(2m<E^2>_{kei}) \tag{54}$$

From (54) the total energy E_3 is related to total momentum p_3 by

$$E_3 = Nm + \frac{p_3^2}{2m} \tag{55}$$

Differentiating (55) leads to

$$dE_3 = \frac{p_3 dp_3}{m} \tag{56}$$

Employing (55) and (56) in (51) then gives

$$N_3 = 2\pi m \int \frac{d\Omega_1 dE_1 dE_2 dE_3 \delta(E_i - E_1 - E_2 - E_3)}{E_3} \tag{57}$$

$E_3 \approx Nm$ and (57) becomes

$$N_3 = \frac{2\pi}{N} \int d\Omega_1 dE_1 dE_2 \tag{58}$$

On the other hand, the scatterers may be strongly coupled to each other, in which case (54) might be replaced by

$$p_3^2 = (\sum_{i=1}^{N} p_i)^2 + 2m N^2 \langle E^2 \rangle_{kei} f \qquad (59)$$

Here f is a function which specifies the degree of correlation of momenta of the scatterers.

The total energy of the scatterers, E_3, is now given by

$$E_3 = Nm + \frac{p_3^2}{2mNf} \qquad (60)$$

$$dE_3 = \frac{p_3 dp_3}{mNf} \qquad (61)$$

(61) and (57) then give

$$N_3 = \frac{2\pi}{f} \int d\Omega_1 dE_1 dE_2 \qquad (62)$$

For a two particle final state, integration of (47) gives first

$$N_2 = \int \frac{d\Omega_1 p_1^2 \, dp_1 \delta(E_f - E_i)}{E_1 E_2} \qquad (63)$$

The total energy $E_f = E_1 + E_2$. (63) is written in terms of E_f and the final state momentum of either particle p, in the center of mass coordinate system. $p = p_1 = p_2$. Suppose both objects are particles with $E^2 = m^2 + p^2$, then

$$\frac{dE_f}{dp} = \frac{dE_1}{dp} + \frac{dE_2}{dp} = p\left(\frac{1}{E_1} + \frac{1}{E_2}\right) \qquad (64)$$

$$N_2 = \int \frac{d\Omega_1 p_1^2}{E_1 E_2} \frac{dp_1}{dE_f} dE_f \delta(E_f - E_i) \qquad (65)$$

(64) and (65) give

$$N_2 = \frac{p}{E_f} \int d\Omega_1 \qquad (66)$$

Suppose now that the ensemble of scatterers is taken as one of the particles satisfying (55). New (56) gives

$$\frac{dE}{dp} = p\left(\frac{1}{m} + \frac{1}{E_2}\right) \approx p\left(\frac{N}{E_1} + \frac{1}{E_2}\right) \tag{67}$$

For N_2, evaluation of (63), then gives

$$N_2 = \frac{p}{(NE_2+E_1)} \int d\Omega_1 \tag{68}$$

If the motion of the scatterers is strongly correlated the integration of (65) will then give

$$N_2 = \frac{p}{fE_f} \int d\Omega_1 \tag{69}$$

It is important to note that our calculations of the phase space factor involve only those parts of phase space for which energy and momentum are strictly conserved. For the appropriate amplitude the only phase shifts which then need to be considered are those resulting from retardation effects to give the phonon or photon field, and the random phase shift of each scatterer associated with the lack of exact equality of the initial and final states.

COHERENT SCATTERING CROSS SECTIONS

Matrix elements have been evaluated for the case where the initial and final scatterer states are identical. As noted earlier, in this case, the cross sections vanish. We now explore situations for neutrino coherent scattering with nearly identical initial and final states.

Electron neutrino interactions are ordinarily described by the object

$$\bar{\psi}_e \Gamma \psi_\nu \bar{\psi}_\nu \Gamma \psi_e \tag{70}$$

$\bar{\psi}_e$ and ψ_e are considered to be field operators ordinarily written as a sum of creation, and annihilation operators for electrons. (70) then results in matrix elements in which one electron is annihilated and another one created. For an N particle initial state there are N matrix elements representing interaction of each scatterer, one at a time. If all these terms are properly phased the sum of the N matrix elements might approach N times the value of each.

The interaction (70) was designed to describe interactions involving small numbers of particles. A similar situation exists for interaction of an electron with the nucleii of a solid. The interaction

$$\bar{\psi}_e(x)\gamma\psi_e(x)D_f(x-y)\bar{\psi}_p(y)\gamma\psi_p(y) \tag{71}$$

could give a sum of terms inolving single protons. In solid state physics the many particle effects are taken into account by employing the lattice vibration normal modes and the electron phonon interaction.

We may interpret the interaction

$$N\bar{\psi}_e \Gamma \psi_\nu \bar{\psi}_\nu \Gamma \psi_e \tag{72}$$

as one involving a direct coupling of N scatterers to one neutrino. The ensemble of scatterers has states of total energy momentum spin. The operators

$\sqrt{N}\,\bar{\psi}_e$ and $\sqrt{N}\,\psi_e$ are expanded in these states of total energy momentum spin. The expansion coefficients are then creation and annihilation operators for the momentum and energy of the ensemble. Under these conditions the integration of (4) would be expected to yield quantities such as

$$N\mu\,\delta(\Delta p_\nu + \sum_i \Delta p_i + p_{ph}) \tag{73}$$

The matrix elements μ are in a certain approximation the same as the ones obtained earlier. The 4 dimensional delta function has as argument the change of neutrino energy momentum, the change of energy momentum of the entire ensemble of scatterers and the momentum of the photon or phenon. An alternative procedure is given by equations (86)-(92).

Suppose now that we can arrange a solid or liquid with a system of quantum states such that coherent scattering of the kind discussed earlier is possible. For electron neutrino scattering the cross section would be

$$d\sigma = N^2\mu^2 E_e E_\nu E_{ph} N_3 \tag{74}$$

with N_3 given by (48), (58) or (62). For photons as one of the final state particles the use of the matrix elements already given leads for the case of scatterers weakly coupled to each other (equation 54)

$$d\sigma = \frac{\alpha G^2 E_e E_\nu dE_\nu dE_{ph} d\Omega K^2 N^2}{(2\pi)^4 m^2 c^4 \hbar^4 c^4} \tag{75}$$

The cross sections will be extremely small unless the ensemble of scatterers can somehow absorb significant amounts of neutrino energy and momentum. If the scatterers behave like a massive, perfectly elastic potential, the scattering cross section would be proportional to the square of

$$\int e^{i\Delta\bar{p}_\nu\cdot\bar{r}}\,\rho_s(r)\,d^3x\,. \tag{76}$$

which is the form factor of the "density" of scatterers $\rho_s(r)$. At MEV energies (76) would give a very small result for a solid. Returning to (75) we imagine first that as a result of internal interactions the ensemble of scatterers can absorb the momentum of the neutrino, but not its energy. Then, the outgoing neutrino might have any value of momentum between its initial value and the negative of its initial 3 momentum. $d\Omega \to 4\pi$, $dE_\nu \approx dE_{ph}$. For strongly coupled scatterers N^2 in (75) might be replaced by a quantity approaching N^3. For (75) as it stands with $N \to 10^{30}$, $dE_{ph} \to 10^{-18}$, vacuum electromagnetic fields, all scatterers in a region small compared with a photon wavelength:

$$\sigma \to 10^{-11} \text{ cm}^2 \tag{77}$$

Suppose a material is employed with scatterers having quantum states which permit most of the neutrino energy momentum to be absorbed. Then each scatterer will have a final state which differs in energy from the initial state by ΔE with

$$\Delta E_s = \frac{E_\nu}{N} \tag{78}$$

and momentum change Δp_s given by

$$\Delta p_s = \frac{E_\nu}{c\sqrt{N}} \tag{79}$$

The phase shifts associated with (78) and (79) are not sufficient to significantly affect the coherence. Under these conditions (75) could be evaluated with

$$dE_\nu \to E_\nu \text{ . For } N \to 10^{30}, \; dE_{ph} \to 10^{-18}$$

$$\sigma \to 10 \text{ cm}^2 \tag{80}$$

For neutrino electron phonon scattering the coherent cross section, assuming (46) is valid, is given by

$$\sigma \equiv \frac{g^2 G^2 N^2 K^2 E_e E_\nu \, d\Omega \, dE_\nu \, dE_{phon} v_s^2}{(2\pi)^4 \hbar^7 c^9} \tag{81}$$

In (81) v_s is the spead of sound. Depending on the internal structure dE_ν again might approach E_ν. The electron phonon coupling constant g is, for metals (CGS units) given by

$$g^2 \rightarrow 10^{-35} \text{erg cm}^3 \tag{82}$$

NEUTRINO INDUCED STIMULATED ABSORPTION

Corresponding to the photon or phonon emission processes there are stimulated absorption processes indicated in the Feynman graphs, figures 3 and 4. The same matrix elements may be employed, but the final state in each instance contains only the neutrino and the assemblage of scatterers. The phase space factors (68) or (69) are appropriate. For the photon case, the cross section for absorption is therefore given by

$$\sigma \approx \frac{\alpha G^2 E_e E_\nu K^2 N^2 \omega_{ph}^2 \, d\Omega}{(2\pi)^4 m^2 c^4 \hbar^2 c^4} \tag{75A}$$

and for the phonon stimulated absorption

$$\sigma \approx \frac{g^2 G^2 N^2 K^2 E_e E_\nu \, d\Omega}{(2\pi)^7 \hbar^5 c^6 v_s} \quad \omega_{pho}^2 \tag{81A}$$

(75A) and (81A) must be summed over all photons or phonons which may be present.

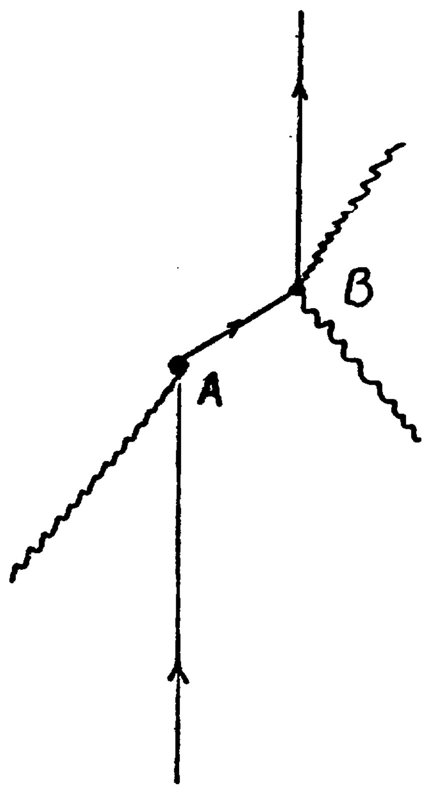

FIGURE 3

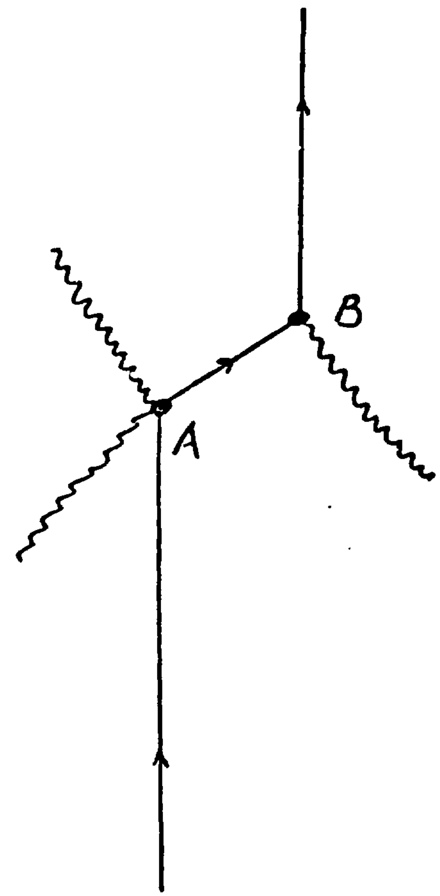

FIGURE 4

CLOSELY SPACED SCATTERER STATE POSSIBILITIES

The importance of having very closely spaced scatterer states is evident. One way of achieving this with electrons is to have each electron exchange energy with a nearby nucleus via the coulomb interaction as indicated by the Feynman graph, Figure 5. We might expect a matrix element having an additional power of the fine structure constant and detailed calculations are being done.

Another possibility consists of employing spin states or some other degree of freedom with quantum state spacing which can be controlled by application of an appropriate field. Suppose that only the temperature of the scatterers is known and therefore that each scatterer quantum state is an appropriate mixture of spin or other states.

It is also assumed that the momentum of the incident neutrino will be absorbed by the assemblage and that only the energy change needs to be accounted for by the possibilities considered here.

Let all scatterers have the same temperature. Let c_n be an annihilation operator for a scatterer with the given mixture of states. Assume that the scattering changes, slightly, the mixture of states. Let c_n' be a creation operator for the new set of states. Each neutrino is imagined to be scattered by the entire ensemble.

Let the initial state be represented by

$$|c_1^+ c_2^+ c_3^+ \cdots c_n^+ | o\rangle \tag{83}$$

and let the final state be one in which all the particles are in a slightly different mixture of states, described by

$$\langle o| c_1' c_2' c_3' \cdots c_n' | \tag{84}$$

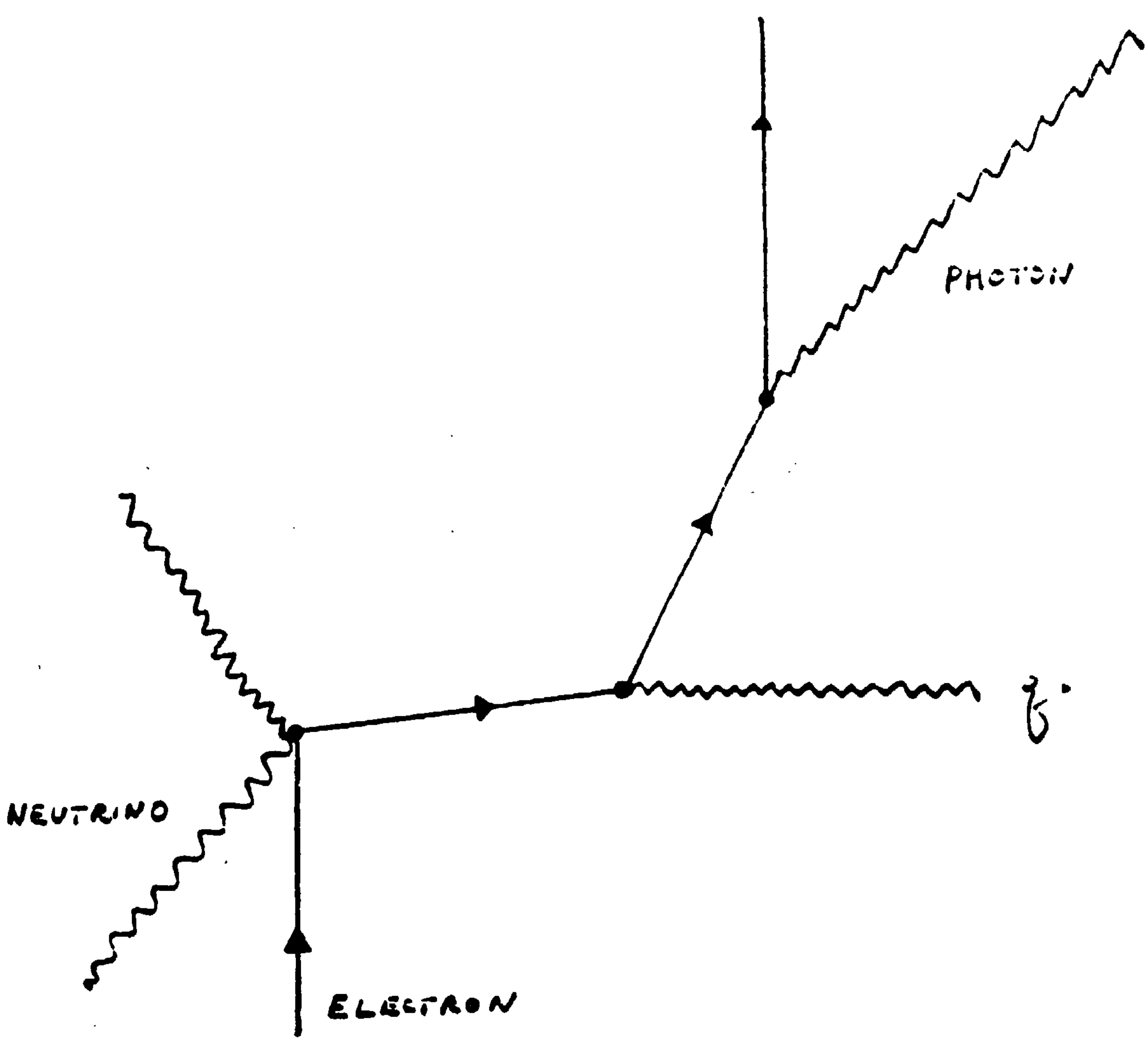

FIGURE 5

For example the state (84) may differ from (83) only in the temperature.

The interaction will then involve the sum $\sum_i \sum_j c_i'^{\dagger} c_j'$. With the initial and final states (84) and (83). The only terms of the interaction which produce coherent scattering are those for which $i = j$ and the summed matrix element is

$$\langle o | c_1' \; c_2' \; c_3' \ldots . c_n' | \; \sum_j c_j'^{\dagger} c_j \; | c_1^{\dagger} c_2^{\dagger} c_3^{\dagger} \ldots . c_n^{\dagger} \; o \rangle \qquad (85)$$

As an example let us evaluate (85) for the case where each scatterer can be one of two spin slates, with c_j associated with the spinor

$$\begin{vmatrix} \beta_j \\ (1-\beta_j^2)^{1/2} \end{vmatrix} \qquad (86)$$

and c_j' associated with the slightly changed spinor

$$\begin{vmatrix} \beta_j - \delta \\ [1 - (\beta_j-\delta)^2]^{1/2} \end{vmatrix} \qquad (87)$$

δ is a very small quantity. (85) may then be evaluated as

$$\langle c_1' \; c_2' \; c_3' \ldots c_n' | \Sigma \; c_j'^{\dagger} c_j | \; c_1^{\dagger} c_2^{\dagger} c_3^{\dagger} \ldots c_n^{\dagger} \rangle = N(\; 1 - \frac{\delta^2}{2} + \cdots + \ldots)^N = Ne^{\frac{-N\delta^2}{2}} \qquad (88)$$

(88) is applicable to all of the earlier processes considered, where the primed states differ from the unprimed ones in the small changes of energy--momentum implied by (78) and (79).

Suppose (88) is applied to spin states in a magnetic field. Let the magnetic moment of each scatterer be μ_m and let a magnetic field H_{MAG} be applied. Then the average change in energy of each scatterer following scattering is

$\Delta \int \psi^\dagger \mu_m H \psi d^3x$ and from (86) and (87)

$$(\Delta E)_{S12} = 4\mu_m H_{MAG} \delta \beta \tag{89}$$

For N scatterers to absorb energy E_ν it is necessary that

$$N(\Delta E)_{S12} = 4\mu_m H_{MAG} \beta \delta \; N = E_\nu \tag{90}$$

to obtain a large cross section (88) requires

$$\frac{N\delta^2}{2} \lesssim 1 \tag{91}$$

The appropriate magnetic field consistent with (90) and (91) is

$$H_{MAG} = \frac{E_\nu}{4\mu_m \beta \sqrt{2N}} \tag{92}$$

where N is the number of scatterers, and δ must remain sufficiently small so that the phase shifts do not destroy the coherence.

DETECTION OF COHERENT SCATTERING BY OBSERVATION OF THE ELECTRIC AND MAGNETIC FIELDS OF THE SCATTERERS

The earlier discussions involved coherent effects in which as a result of neutrino excitation an entire ensemble of scatterers emitted one photon or one phonon which is detected. It appears possible to obtain larger coherent cross sections by omitting the step of photon or phonon emission. The coherent effect appears observable by making use of the Coulomb type electric or magnetic fields of the scatterers. This process appears applicable to electrons, protons, neutrinos, and certain nuclei .

Without the photon emission, the matrix element (4) for electron neutrino scattering is

$$\mu_{e\nu} = \frac{G}{\sqrt{2}} \sum_{\text{electrons}} \left[\bar{U}_{ef} \gamma^{\alpha} (1+\gamma_5) U_{\nu i} \bar{U}_{\nu f} \gamma_{\alpha} (1+\gamma_5) U_{ei} \right] \tag{93}$$

and for either proton or neutron - neutrino scattering

$$\mu_{n\nu\nu} = \frac{G}{\sqrt{2}} \sum_{\text{Protons-neutrons}} \bar{U}_{nuf} \mathcal{H} (U_{\nu}, \bar{U}_{\nu}, g, \gamma_{\mu} \ldots.) U_{nui} \tag{94}$$

where the object $\mathcal{H}$ is a function of neutrino creation and annihilation operators, form factors and gamma matrices. The objects (93) and (94) are summed over all scatterers and then squared. This squared sum, in ordinary scattering experiments is approximately N times each squared term. The scattering is incoherent because the initial and final states are usually very different and have random phases. Coherence effects again appear possible if the initial and final states are nearly, but not exactly, the same.

Let each scatterer be imagined localized near some site in a solid or liquid and to have a number of closely spaced quantum states. Let each scatterer be in an appropriate mixture of such states, defined for example by the temperature and a density matrix. Since all scatterers so far considered have spin $\neq$ o the treatment similar to (83)-(92) appears applicable.

The scattering cross section for such a process is evaluated as approximately

$$\sigma = \frac{G^2 E_s E_\nu N^2}{(2\pi)^4 \hbar^4 c^4} \tag{95}$$

In (95) E_s is the energy of each scatterer, electron or nucleon.

Each scattering produces a change δ in the spin state mixture with the magnetic field so arranged that the scattering produces an absorption of energy E_ν by the spin system. If the spin system is totally isolated from the lattice, the degree of alignment of the spins would continue to increase until all moments are antiparallel to the magnetic field. However, as soon as β starts to change from its thermal equilibrium value the spin system begins to relax to the lattice temperature. A stationary state may ultimately be reached corresponding to a spin temperature different from the lattice temperature. It is proposed to observe this change in spin temperature.

The orientation energy of the spin system in the external field U_{Spin} is given by $\int \psi^\dagger \mu_m \; H_{mag} \psi \, d^3x$, as

$$U_{Spin} = N H_{mag} \mu_{mag} (2\beta^2 - 1) \tag{96}$$

$$\frac{dU_{SPIN}}{dt} = 4 N H_{mag} \mu_{mag} \frac{d\beta}{dt} \tag{97}$$

This rate of change of energy is determined by the absorption of energy from the neutrinos with a rate governed by the neutrino flux F, and cross section σ, the spin-lattice transition probability ω_{12} from the lower to the upper spin state and ω_{21} from the upper to the lower spin state. Therefore, we have from (96) and (97)

$$4\, NH_{mag}\mu_{mag}\frac{d\beta}{dt} = \sigma FE_{\nu} + (1-\beta^2)N\omega_{21}\Delta E - \beta^2 N\omega_{12}\Delta E \tag{98}$$

ΔE is the pure spin state energy difference, not the difference in mixed state energies.

In the absence of neutrinos the steady state value of β for which we employ the symbol β_o is given by

$$(1-\beta_o^2)N\omega_{21} - \beta_o^2 N\omega_{12} = 0 \tag{99}$$

If the neutrino scattering produces a total change $\Delta\beta$ when a stationary state is reached, we have from (98) and (99)

$$\sigma FE_{\nu} - [(2\beta\Delta\beta)N\omega_{21} + 2\beta\Delta\beta N\omega_{12})\Delta E] = 0 \tag{100}$$

$\omega_{12} \approx \omega_{21}$ if $\frac{\Delta E}{kT} \ll 1$ and for this condition

$$\Delta\beta = \frac{\sigma FE_{\nu}}{4\beta N\omega_{21}\,\Delta E} \tag{101}$$

In the absence of scattering

$$\frac{\beta^2}{1-\beta^2} = e^{-\frac{\delta E}{kT}} \tag{102}$$

In (102) δE is the difference in energy between the two states $\delta E = \frac{E_\nu}{\sqrt{N}}$. Differentiating (102) leads to

$$\frac{(d\beta)_T}{(1-\beta^2)\beta} = \frac{\delta E}{2\ kT} \quad \frac{dT}{T} \tag{103}$$

with $(d\beta)_T$ the change in β associated with a change in temperature dT. The steady state change due to scattering (103) is thus observable as an apparent difference in spin and lattice temperatures $(dT)_{equivalent}$ with

$$\frac{dT_{equivalent}}{T} \quad \frac{\delta E}{2kT} = \frac{\sigma F E_\nu}{4\beta^2 N\omega_{21}(1-\beta^2)\delta E} \tag{104}$$

Since $\delta E = \frac{E_\nu}{\sqrt{N}}$, (104) may be written in the form

$$\frac{dT_{equivalent}}{T} = \frac{\sigma F}{2\beta^2\omega_{21}\ E_\nu} \quad (kT) \tag{105}$$

ω_{21} is the reciprocal of the spin lattice relaxation time. It is known that for some materials -- for example lithium fluoride, the spin lattice relaxation time is several minutes. If the cross section (95) can be achieved, 10^{22} nuclear spins will give a coherent cross section $\sigma = 1\ cm^2$.

For T=300°K a neutrino flux 10^5 per square centimeter per second at 1 MEV would double the spin temperature. Lowering the temperature will reduce ω_{21}.

There are a number of other possibilities. Nucleons might be endowed with spin greater than 1/2, and have quadrupole moments. An example is deuterium.

Closely spaced states result from the interaction of the quadrupole moment with the internal crystal field. Another method would employ quantum state mixtures which have an electric dipole moment and produce an appropriate structure by application of an electric field.

COHERENT SCATTERING BY A CRYSTAL

The following example has been instructive in understanding what may be important factors in obtaining a large cross section. Consider a crystal containing a large number of scatterers. Let the crystal recoil and absorb whatever momentum must be exchanged. The total mass of the scatterers is so large that the transfer of momentum from the neutrino will not be accompanied by significant energy transfer. The neutrino energy may be transferred by change of the scatterer spin state mixture or by other means. The crystal is assumed to be very stiff.

Consider the S matrix element involving neutral currents

$$S = \int \bar{\psi}_{sf}\, \Gamma\, \psi_{so}\, \bar{\psi}_{\nu f}\, K\, \psi_{\nu o}\, d^4x \tag{106}$$

In (106) the subscripts S and ν refer to scatterer and neutrino respectively. The subscript o again refers to the initial state and the subscript f refers to the final state. Γ and K are the required operators for the neutral current interactions.

The assumption of stiffness is interpreted to mean that the final state of the scatterer has the same particle density distribution

as the initial state. Therefore

$$\psi^*_{sf}\,\psi_{sf} = \psi^*_{so}\,\psi_{so} \tag{107}$$

the final state wavefunction for the ensemble of scatterers is therefore

$$\psi_{sf} = \psi_{so}\, e^{-i\Delta E_s t + i\Delta\bar{p}_s\cdot\bar{x}} \tag{108}$$

ψ_{sf} has the properties

$$\int \psi^*_{sf}\,\bar{p}\,\psi_{sf}\,d^3x = \int \psi^*_{so}\,\bar{p}\,\psi_{so}\,d^3x + \Delta\bar{p}_s \tag{109}$$

$$\int \psi^*_{sf}\, i\frac{d}{dt}\,\psi_{sf}\,d^3x = \int \psi^*_{so}\, i\frac{d}{dt}\,\psi_{so}\,d^3x + \Delta E_s \tag{110}$$

The incident and scattered particles are represented by plane waves. The S matrix (106) may then be written employing (107) as

$$S = \int \bar{\psi}_{so}\,\Gamma\,\psi_{so}\,\bar{U}_{\nu f}\,K\,U_{\nu o}\, e^{i(\Delta p_\nu + \Delta p_s)_\mu x^\mu}\, d^4x \tag{111}$$

Let us define a Fourier transform $\phi(p_t)$ by the relation

$$\phi(p_t) = \int \bar{\psi}_{so}\,\Gamma\,\psi_{so}\,\bar{U}_{\nu f}\,K\,U_{\nu o}\, e^{-i(p_t)_\mu x^\mu}\, d^4x \tag{112}$$

Employing (112) in (111) then gives

$$S = \int \phi(p_t) e^{i(p_t + \Delta p_\nu + \Delta p_s)_\mu x^\mu} \, d^4x \, d^4p \tag{113}$$

(113) is evaluated as

$$S = \phi(-\Delta p_\nu - \Delta p_s) \tag{114}$$

Let us evaluate (114) for an ensemble of scatterers in a stiff crystal with the n^{th} scatterer confined to a cubical box $\Delta\ell$ units in length at space coordinate x_n. The result is

$$S = \bar{U}_s \Gamma U_s \bar{U}_{\nu f} K U_{\nu o} \left[\sum_{n=1}^{n=N^{1/3}} \frac{e^{i(\Delta p_\nu + \Delta p_s)_x X_{nx}} \sin\left[\frac{(\Delta p_\nu + \Delta p_s)_x \Delta\ell}{2}\right]}{\frac{(\Delta p_\nu + \Delta p_s)_x \Delta\ell}{2}} \times \right.$$

$$\sum_{n=1}^{n=N^{1/3}} \frac{e^{i(\Delta p_\nu + \Delta p_s)_y X_{ny}} \sin\left[\frac{(\Delta p_\nu + \Delta p_s)_y \Delta\ell}{2}\right]}{\frac{(\Delta p_\nu + \Delta p_s)_y \Delta\ell}{2}} \times$$

$$\left. \sum_{n=1}^{n=N^{1/3}} \frac{e^{i(\Delta p_\nu + \Delta p_s)_z X_{nz}} \sin\left[\frac{(\Delta p_\nu + \Delta p_s)_z \Delta\ell}{2}\right]}{\frac{(\Delta p_\nu + \Delta p_s)_z \Delta\ell}{2}} \right] \int_{-\infty}^{\infty} e^{i[\Delta E_\nu + \Delta E_s]t} \, dt \tag{115}$$

(115) is identified as

$$S = N\bar{U}_s \Gamma U_s \bar{U}_{\nu f} K U_{\nu o} \delta_4(\Delta p_\nu + \Delta p_s) \tag{116}$$

(115) would give the form factor of the solid if Δp_s and ΔE_s were not present. The exchange of energy and momentum in a controlled way is very essential. For N scatterers the total cross section might then approach the value

$$\sigma \rightarrow \frac{G^2 N^2 E_s E_\nu \mu^2}{(2\pi)^4 \hbar^4 c^4} \tag{117}$$

EXPERIMENTS

The theory already presented suggests that coherent scattering might be observed, with large total cross sections for the scattering processes Figures 1 and 2. In these a meutrino is scatte . by an electron and a low frequency photon is emitted, with cross section given by (75). The earth's magnetic field satisfies the require-ments of (92).

One series of experiments was carried out employing a large tank of demineralized water, four feet in diameter and 4 feet long. A second series employed manganese nitrate in a chamber 20 inches in diameter and 20 inches long. The reactor at the National Bureau of Standards in Gaithersburg, Maryland was employed. It has a power output of ten megawatts and operates continuously except for maintenance periods. The apparatus was located in a well shielded area about thirty feet from the core. Search was carried out in the band 25-50 Mcs, and in another series, in the band 4-1000 Mcs. Small effects were seen. However the power received for the 25-50 Mcs experiments was less than 10^{-14} watts, and for the 4-1000 Mcs experiments the power receive was less that $4x10^{-13}$ watts.

The Manganese nitrate experiment was repeated at the A.F.R.R.I. Triga Reactor. The pulsed mode was employed, with 1660 megawatt 10 millisecond pulses. The high neutron flux levels and the pulsed mode made observations difficult and it was decided to return to the National Bureau of Standards reactor.

At the National Bureau of Standards reactor there are many experiments which require a constant power level. The reactor is turned off about every forty days for maintenance. The most useful observing times are when the reactor is being switched on and off. The electron scattering experiments were observing effects too small to be of interest for the major research objectives. Extremely long running times would be necessary to verify that the effects were not due to st .is-tical fluctuations. It was decided to explore the neutral current scattering processes discussed in equations 93 - 117.

First a teflon cylinder 0.75 inches in diameter and 2 inches long was employed. The 50 kilowatt reactor at the University of California Irvine was available. The apparatus was approximately 50 feet from the core. The teflon was at room temperature and an increase in temperature was searched for. Results were negative.

The experiment was repeated at the ten megawatt National Bureau of Standards reactor and again results were negative. In these experiments a power output of 3000 ergs per second could have been observed as a heating effect.

It was then decided to employ the Al^{27} nucleii in a sapphire crystal, again 0.75 inches in diameter and 2 inches long. According to the theory discussed in equations 106 and 107 sapphire should be have a much larger total scattering cross section than teflon. Very small effects were suspected at room temperatures. It was then decided to repeat the experiments in a glass Dewar at liquid helium temperatures. Under these conditions a very small heating effect is much easier to observe.

A new series of experiments, beginning in June 1978 appears to give a strong positive result. It is observed that 0.15±.03 ergs per second are generated within the sapphire target as the reactor is switched on.

DISCUSSION

A research reactor area nearly always has a very high level of human and apparatus activity at times when the reactor is being shut down or starting up. Many important checks are required, for the reactor and for continuing experiments near the reactor. Large pumps are switched and large power fluctuations may occur.

Exhaustive checks were carried out to determine if the activity unrelated to the neutrino flux could be causing the observed effects. In addition radiation surveys have been carried out in the vicinity of the sapphire apparatus. The gamma and neutron radiation levels are roughly .1 millirems per hour. None of these effects appear capable of producing the 0.15 erg per second heating effect.

Therefore it appears possible that the large cross section characteristic of the coherent process of equations 93 - 117 is being observed. This conclusion must be regarded as tentative and uncertain, until additional checks can be carried out.

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